The Two Revolutions, Land Elites and Education during the Industrial Revolution

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Abstract: The understanding of the Industrial Revolution, the process of transition from a Malthusian equilibrium to today's Modern Economic Growth, has been subject to a passionate debate. This paper adds more insights on the process of industrialization and the demographic transition that followed this period. By applying the theory of interest groups to landownership and by analyzing land elites' incentives to allow education, it is showed that the political power of land elites is important for understanding the main events during Industrial Revolution. Besides, additional contributions are made on the existence and role of the Agricultural Revolution and initial land fertility to the process of industrialization. It is showed that Agricultural Revolution played a significant role on fastening the process of industrialization and land fertility may, as well, had a positive effect. A model and numerical simulations are presented to show these results.

Keywords: Industrial and Agricultural Revolution; Demographic Transition; Education; Interest Groups.

Jel Classification: N53, O13, O14, O43, O50

1. Introduction

The Great Divergence, which started two centuries ago, has been one of the main research challenges economists have been facing in growth and development fields of study. The understanding of the Industrial Revolution has been subject to a passionate debate. Many hypotheses have been put forward to explain the process of transition from a Malthusian era to a Post Malthusian and today's Modern Economic Growth era. Unified Growth Theory has attempted to understand and put forward explanations on the behavior of the economies in this particular time period. Comparative economic development has considered factors such as geographical, institutional, ethnic, religious, human capital formation and colonization as main explanatory elements. Meanwhile, the processes of declining
fertility, educational and human capital formation, and the agricultural transformation were intimately related with the onset of the Industrial Revolution as Unified Growth Theory has consistently showed (Galor and Weil, 2000; Galor and Moav, 2004; 2006; Voigtländer and Voth, 2006).

In this paper, by applying tools from Unified Growth Theory and the theory of interest groups, it is sustained that the way land elites (landlords) observe their gains and losses from the process of education and decide to support or not education change the ability and willingness of workers to educate their children and, hence, the timing of the provision of education among population. This force preventing the rise of education will, on one hand, delay the process of demographic transition and, on the other hand, delay the real take off of the industrial sector that cannot fully achieve its full potential without a major level of education. Indeed, the lack of institutions that promote human capital (public schooling, child labor regulations and other time reducing cost institutions) reduces the rate and the timing of the transition from an agricultural to an industrial economy. Along with this argument, it is also proposed that the improvement in the agricultural processes, which is claimed to have happened in the century previous to the Industrial Revolution, have contributed to accelerate the process of industrialization. In the same way, it is argued that it also had contributed to a more favorable decision on education by land elites since the risk of losing rents and their share in the economy is lower because of the prevailing higher productivity of land. By the same token the role of initial land fertility, which is examined as a byproduct of the present model, has a positive effect on education as well, although it goes against the conclusions of other existing studies (Engerman and Sokoloff, 2000; Galor et al, 2009; Litina, 2012).

The Industrial Revolution as a process of transformation of an agricultural economy to an industrial one had a different timing on different countries. This different timing concerning both the take-off and the demographic transition led to the so called “Great Divergence” in income per capita as well as on population growth across regions. Although in the end of the first millennium Asia was the world leader in both wealth and knowledge, in 1800s Europe had already surpassed those societies (Pomeranz, 2000; Galor, 2011). Empirical analysis on this period and onwards shows that besides England, where Industrial Revolution
first took place, most of countries in continental Europe followed the trend and had their own process of industrialization. France, Belgium, Prussia and the Netherlands are some examples of western countries that witnessed this revolution just after England where this process starts earlier (Bairoch, 1982). Along with these countries, the Western offshoots, such as the US, Canada and Australia also witnessed the development of their economies, sooner surpassing the European countries. As for all other countries in the world, most of their economies remained stagnant for almost the last two centuries (Landes, 1998; Maddison, 2003).

In the late eighteenth century a new social and economic phase – the Post-Malthusian period – began. It differs from the last by breaking the stagnating equilibrium and by preparing the ground for the transition to the Modern Growth regime. Maddison (2003) identifies that both population and income per capita start to increase simultaneously in this period (early 1800s) - the previous Malthusian checks to income per capita no longer exist. Moreover, fertility rates continue to increase even more until the middle 1800s (Dyson and Murphy, 1985; Lee, 2003). Along with these demographic trends, another important feature is the continuous and progressive process of industrialization. As these forces start to be pervasive in the western countries, the Malthusian trap becomes less and less powerful and with the rise of demand for human capital this trap is overcome definitely (Galor, 2011).

Eventually, as these processes of education and demographic transition occurred, the onset of a Modern Growth era arrived. Following this decline on fertility and the rise of education and, hence, human capital formation, income per capita increases consistently over the years until nowadays. As it is observed today in all developed countries, these trends persist. They are even more accentuated in countries where Industrial Revolution occurred in the nineteenth century while in countries in Latin American and Asia it occurred in the middle twentieth century, while in Africa was the last place it started.

Concomitant in this period of transition from a Malthusian era to Modern Growth, three features excel: first, the Agricultural Revolution had an impact on the eighteenth century economies, namely on England; the education dependence on the willingness and support of elites, state and government and, finally, the
demographic transition which derived from the quantity - quality trade-off. These features, which will be addressed in the next section, are essential to reconcile the events during, and after, the Industrial Revolution period and how they are interconnected. Why did the education lag remain for so long? Did the Agricultural Revolution contributed to the onset of the Industrial Revolution and education of population? If so, to what extent? Can the forces behind these developments be uncovered? Did they fasten or delay the spread of education? Can elites’ decisions under this period be modeled?

This paper aims to show that elites only financially support education if that is profitable to them. While other approaches suggest that education would mostly harm land elites by minimizing their rents (Galor et al., 2009), we suggest that it indeed may harm initially but, as time goes by, it will be profitable to tax and provide education. Provided that gains on rents, due to the slower growth of land productivity and the increasing marginal spillover gains from the industrial sector, become higher than the cost of taxes, elites will have an economic incentive to allow and abide to give funds to education. Therefore, one of the main novelties of this paper is to explore the predictions and the validity of an alternative but complementary explanation for the rise of education where land elites agree on the provision of education. While in most of literature it is always agreed that they are against these measures but in the case of small landowners (Galor et al., 2009), this paper sustains there is a threshold and a time when they, decide to support education of population. The fact that elites can be better off with taxation is an interesting conclusion and clears the systematic negativist perspective on elites. It also provides an alternative explanation for the education time lag since it attributes to the decisive political power of elites to determine when education is provided, although it will keep the quality level on the hands of population, allowing for the quantity-quality trade-off to still be a characteristic of this era.

In addition, and following the patterns during the transition from the Malthusian era to the Modern Growth era, it is argued that the process of Agricultural Revolution and natural fertility of land have an impact on the way landlords agree with the process of education of population. In fact, it is showed

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1 It will be assumed that elites provide financial support by dispensing part of their wealth in the form of a tax on their wealth.
that continuous technology advancements make regions, and even countries, allow education to emerge sooner. So, a contribution can be added to the debate on the positive and negative impacts of the Agricultural Revolution on the Industrial Revolution, and discuss when did it actually occurred. Indeed, it must be asked how the rise of productivity of land may have induced a negative and a positive effect for the effectiveness of the Industrial Revolution. The first effect, relates to the higher marginal gains for farm workers compared to urban workers, contributing to a lower pace of migration from the countryside to urban areas. The second one relates with the willingness of elites to provide education. Since their rents increase to higher levels due to a more productive labor force, the loss driven from the financial support to educate population and the loss of workers to the industrial sector become less significant. The risk of being overtaken by industry vanishes sooner since the benefits of externalities from industry become bigger than the prejudices of allowing industry to complete its take-off at an early phase. Although elites do not immediately allow for education, the necessary incentives arrive earlier since agricultural technology in the Industrial Revolution is already more developed.² It will be also explored the hypothesis of how initial land fertility affects elites’ decisions. Again, by the same token of Agricultural Revolution, a more favorable land endowment causes productivity to be higher in the agricultural sector and, therefore, elites find less risky to allow education to emerge at an earlier stage. This conclusion runs against some of the existing literature (e.g. Engerman and Sokoloff, 2000; Litina, 2012) but it is assumed that this result is more a byproduct of the model than a main result. It represents a positive force among all the existing forces land fertility imposes.

To sum up, the proposed argument aims to add to the existent literature a distinctive explanation for the delay of education and consequent demographic transition and the differences among countries on its emerging timing, always including the Malthusian, Post-Malthusian and Modern Growth trends inherent to this time period, by proposing a complementary perspective of land elites under Unified Growth Theory. Finally, it contributes to the discussion on the positive and

² Besides these effects, we have also the positive effects highlighted in the literature on the provision of more and cheaper goods as well as migration of labor to urban population which would sustain and expand it (Overton, 1996; Allen, 2009).
negative impacts to elites’ decisions and, consequently, on education, of Agricultural Revolution and land fertility.

The paper is organized as follows. In section 2, we present a historical overview of the periods before and through the Industrial Revolution period. Some related literature is presented in section 3. In section 4, the set-up of the model is defined as well as the main assumptions. Section 5 provides the analysis of the main predictions of the model and a discussion is drawn on the main results. Finally, some concluding remarks are made in the closing section.

2. Historical Overview

The divergence that began in the Industrial Revolution delimited the end of the Malthusian era in a path towards the Modern Growth regime. The Malthusian era was characterized by a continuous struggle of population for survival. Due to the interconnections between technology and population, only breakthroughs of technology could lead to temporary income per capita gains. Improvements of technology initially had a positive effect on productivity and, hence, on income but ultimately these gains would vanish because the rise of fertility. In a nutshell, any technology gains were in this period channeled to population growth while income per capita remained almost constant, resulting in a stagnant economic environment. Therefore, income per capita during all the period between 1 AC to 1500 AC was kept almost constant (Maddison, 2003).

Besides this, during this period other forces were said to be influencing the economy mainly in England. What was later called Agricultural Revolution, is supposed to have led to the improvement of agricultural productivity and, hence, standards of living of population. The importance of Agricultural Revolution in the creation of the modern world economy is, for some, greater than the Industrial Revolution itself. Indeed, between 1700 to 1850, it made possible that output per acre and output per worker increase to levels far from those verified during the medieval ages (Clark, 1993). There is quite controversy on the real dimension of the Agricultural Revolution and if it really occurred in England. It is argued that way before the Industrial Revolution English farmers were already quite productive (Mokyr, 2009).
Looking at the estimative of rents in different studies for England, we observe that the usual results present an upsurge of rents in the beginning of the 17th century and then there is a slow motion on the growth of rents until the beginning of the 19th century (Allen, 1988; Clark, 2002). Nevertheless, there were some significant changes in the English process of farming that amounted for a rise in the output and productivity of land. Two levels are considered to allow for the increase of output: intensity on the usage of land and efficiency on its usage (Brown, 1991; Clark, 1993; Mokyr, 2009). On the efficiency gains we observe during this period a change in the crops used on plantations: more productive ones and with higher value. A new rotation system, more intensive and less restricted on the selection of land “in rest”, was adopted by farmers. The agricultural knowledge also spread across the country with the adoption of new tools and new methods of farming. The enclosure process that started already at the beginning of the 18th century had a decisive impact on the economy in the late 18th century early 19th century.

On the expansion of cultivated area, we observe during this period an increase of arable land and pasture land due to the process of enclosure on one side and due to the decrease of “wasted land” – land that was neither used for grazing nor for farming. Arable land increased from 11 million to 14.6 million acres and pasture increased from 10 to 16 million acres in the period between 1700 and 1850 (Brown, 1991; Mokyr, 2009). At the same time the cultivated area increased also its quality - new farming methods were introduced. New crops were introduced in the new land taken from waste or fallow. Only the best seeds were selected to use on land. Only the animals with the best reproductive characteristics were chosen to breed and some species were brought from abroad to improve livestock cattle. This had a great impact by increasing the productivity of land and also the supply of food. New rotation systems were also introduced adding to the boost of production. The improvement of soil fertility was also accomplished by the increase of usage of manure and other natural fertilizers, such as marl and lime. In addition, water meadows were very common as well as the drainage of fens (Brown, 1991; Allen, 2009).

Finally, the enclosure process, which took place mostly in England, has been a very controversial subject since there is an ongoing discussion on its real effects
previous to the Industrial Revolution (McCloskey, 1972; Clark, 1993; Allen, 2009). The process of enclosure of open fields started in the mid 1500s and until 1700 half of the cultivated had already been enclosed. In the late 1700s there was a boom on enclosure and the last lands were enclosed (Brown, 1991). The goal was to achieve higher productivity due to a better organization of lands, easier agreements on new production techniques, and an increase on the size of the average agricultural holding (Mokyr, 2009). Instead, the old open fields were divided between several cultivators and each cultivator had its individual right to cultivate his share of land until the harvest. Although this previous land organization was said to be one of the main causes of land lower productivity, Allen (2009) asserts that innovative methods were also introduced in open field land and McCloskey (1972) claims that most of the fields suffered improvements and technology progress during the period of 1700 to 1850.

Education was already regarded as an asset in the eighteenth century although it had a minor role during the first phase of industrialization. Only after the middle 1800's, when demand for education was reaching a fever pitch, education starts to rise and become essential for the definite take-off of the industrial sector. In the first phase of industrialization demand for skilled workers was small, because the requirements to work on industry were still very simple - illiteracy was still very common among workers. As industrialization moved on, working in industry became more and more demanding and a higher level of education was required. Despite educational reforms were taking place during the eighteenth and nineteenth centuries, the most important ones, which led to a real increase in the educational level of people, only emerged in the late nineteenth century. This was pernicious for the economy since despite the high demand for education and capital formation, each country had his own pace on placing educational reforms (Cubberley, 1920; Galor, 2011). Laws regarding schools and education started to emerge in several countries in Europe already in the eighteenth century. In Prussia, the first laws regarding the establishment and organizing schools were issued in the early 1700s. Many measures were taken to oblige the attendance to schools for the children. These measures met with resistance everywhere since there was no acceptance by population in general, and landlords in particular, to cope with the financial burden. This conducted to a slow advance of the measures
that would be expected to be taken. Only after, in the middle nineteenth century and with state intervention did the reorganization of the educational system from elementary school to university became effective (Cubberley, 1920).

The same happened in France and Italy, where the influence of the French revolution, and the new tendencies on education changed the way of education was envisaged. Starting with the substitution of church schools to state schools, small steps were given during Napoleon’s period. But after his fall and the fall of the following monarchy, education regressed by the imposition of restrictions to state schools and the enhancement of church schools and private schools that promoted a more favorable teaching for the dictatorship by the time. Only later education regained its previous position on society’s priorities (Cubberley, 1920; Green, 1990).

As for England, the process of education started only in the 1850’s when several reforms were effective in promoting education among children. After dominating the industrial field since the beginning of the Industrial Revolution, England began to fell behind to continental countries and so after the 1850’s several acts were approved to try to retake the leadership in manufacturing technology (Flora et al., 1983; Green, 1990).

Besides education, another key trend was emerging in this period: the decline of fertility rates. This decline characterized the demographic transition in most countries along the last two centuries. In Western countries this reduction of population growth started in the late nineteenth century while in Latin America and Asia this phenomenon began much later in the middle twentieth century. This transition has continued through the last century and has contributed to fertility reach the limit of the replacement level (Lee, 2003).

More interestingly is how several studies show that education and fertility are interconnected. The decline of fertility was dominated by investment on education so that there was a negative correlation between both factors (Flora et al., 1983). This negative correlation is associated in several studies with the trade-off between child quantity and quality. Becker et al. (2010) and Becker et al. (2012) found evidence of this trade-off in the nineteenth century Prussia while Murphy (2010) finds evidence for France in the late nineteenth century. If this is true, any
explanation of the transition during the Industrial Revolution must account for this phenomenon.

Some authors have shown that small interest groups promote a blockage of new technologies and better institutions in order to keep their own power and their rent extraction. As Mancur Olson teach us: “...small groups in a society will usually have more lobbying and cartelistic power per capita...” (pp. 41, Olson, 1982). Indeed, small groups organize to pursue their own interest disregarding society’s interests as a whole, blocking and delaying any process of development and the shift of institutional or technological environment when it does not suit their interests (Olson, 1982; Acemoglu and Robinson, 2000; Lizzeri and Persico, 2004; Acemoglu and Robinson, 2008)

The period of Industrial Revolution was no exception for the rise of these groups. Land elites were a small group in pre-industrial societies. They were too powerful and their initial incentives were of halting any process of education, and, hence, the complete take-off of the Industrial Revolution (Galor et al, 2009). As referred before, this group was the one the state recurred to finance education. This power and unwillingness to support education was the main reason for the conflict between the emerging capitalist class and the old landowners. In fact, the transition from an agricultural to an industrial economy has changed the pervasive agrarian economy conflict to the industrial conflict. While the former had the landlords and the masses as main protagonists, in the latter the competition for power was taken between these land elites and the emerging industrialist elites. The fight for more education in these centuries was one of the main points of divergence between these two groups. While industrialists wanted more educated masses to boost their production, land elites would perceive the loss of land workers to cities and so opposed determinately to educate them. The power of these elites on this period of time was strong enough to prevent the dissemination of education. The financial influence of land elites, the big and richest group by the time implied that most of kings and princes’ decisions depended on the own advantages elites could have (Ekelund and Tollison, 1997; Lizzeri and Persico, 2004). In fact, the dependence of kings on landlords’ money for warfare and other expenses made it easy for the latter to impose to the king the concession of monopolies, private businesses, patents, and other advantageous businesses,
where it could be included a less disseminated but public education. For instance, it is known that, deriving from the mercantilist era, the power of the state was, in this time, flooded with private interests and interest groups that managed efforts to conduce policies in the way it suited them most (Ekelund and Tollison, 1997).

3. Related Literature

The role of institutional factors has been studied as main determinants of economic performance and, hence, of the great divergence. Having a higher acknowledgement in the last decades, modern institutional theories had their initial historical birth with North and Thomas (1973), North (1981), Greif (1989), North (1990), and were followed in a more empirical fashion by La Porta et al. (1999), Rodrik et al. (2004), Banerjee and Iyer (2005), Hall and Jones (2010) and others. Institutions are stressed as having the force to shape and guide agents in their actions. Authors defend institutions such as property rights, informal laws, culture, contracts, and so on can facilitate the enhancement of economic growth since they constitute a myriad of rules, both formal and informal, that constitute the framework in which agents behave. For instance, the forces of environment can influence the evolution of the economy indirectly by determining how societies build their rules - institutions (Engerman and Sokoloff, 2000; Acemoglu et al, 2001; Acemoglu and Johnson, 2005; Nugent and Robinson, 2010). According to this theory, the natural land endowments were a source of more, or less, inequality, which eventually triggered extractive institutions. The kind of land was determinant to the type of institutions generated in each region.

Under the same branch, the processes of political and social conflict were examined by many authors Mancur Olson (1982), Mokyr (1990), Acemoglu and Robinson (2000), Bourguignon and Verdier (2000), Grossman and Kim (2003), Lizzeri and Persico (2004), Acemoglu and Robinson (2006) among others. Some argue small interest groups, the elites, have the power to influence the distribution of income among population, and may collude to defend their own private interests. Others consider this effect but highlight that elites may support reforms and redistribution to the masses to avoid socio-political instability and predation, stimulating investment and economic growth. For example, on the effect of elites to education, Bourguignon and Verdier (2000) argue that if education determines
political participation, it may happen that elites do not find beneficial to subsidize universal public education although there are positive externalities from human capital. While Grossman and Kim (2003) show that predation is mitigated by education, Lizzeri and Persico (2004) argue that elites use the provision of public services to their interests, so that the extension of franchise redirects resources from wasteful redistribution to public goods. As for Doepke and Zilibotti (2005) show that child labor regulation may benefit capitalists by increasing human capital of workforce through education of children.

Under this strand, the role of human-capital formation is highlighted as a key element to the transition from stagnation to growth. This line of research is elegantly built by Unified Growth Theory (Galor, 2011). This theory links the rise in the demand for human capital in the emergence of industrialization with technological progress, and the onset of demographic transition, conducting to the transition from stagnation to growth (Galor and Weil, 2000). Further research has studied human capital in the light of the dispute between elites. Galor et al. (2009) suggest that the importance of human capital in production increased the incentives for capitalists to support the provision of public education triggering the demise of the existing class structure whereas Galor and Moav (2006) argued that inequality in the distribution of landownership negatively affected the emergence of human-capital promoting institutions. It is to this line of research that this paper contributes with an analysis of the willingness of land elites to support education during the Industrial Revolution and taking also into account the previous process of the Agricultural Revolution.

Other factors have been emphasized as determinants for economic growth and the great divergence. For instance, geographical factors suggest that more favorable geographical conditions made Europe less vulnerable to the risk associated with climate and diseases, leading to the early European take-off (Jones, 1981; Diamond, 1997). While cultural/institutional factors imply that societies where norms and ethics enhance the "entrepreneurial spirit" and the openness to new ideas and advances in other societies (cultural assimilation) are the ones that attained industrialization during the Industrial Revolution period 1800’s (Hall, 1986; Landes, 1998; Landes, 2006; Ashraf and Galor, 2011). Nevertheless, the geographical and cultural factors are out of the scope of this paper.
4. Model Setup

Along with Galor et al. (2009), Ashraf and Galor (2011) and Litina (2012), consider an overlapping-generations economy operating over infinite discrete time. In the pre-industrial era, the economy produces a single homogeneous good, using land and labor as inputs. After the emergence of the industrial sector, the economy produces agricultural and manufactured goods, using as inputs land and labor. The supply of land is exogenous and fixed over time. The number of efficiency units of labor is determined by households’ decisions in the preceding period regarding the number and human-capital level of their children.

The model comprises two types of individuals: workers and elites. Workers reproduce themselves asexually so that each individual has a single parent. The number of offspring relies on workers’ decisions. In turn, elites have one child each. They own land, consume, leave a bequest to their child and use power to set taxes. Their income comes only from land rents and the bequest left by their parents. Thus, in each period $t$, a generation of a continuum of $L_t$ identical workers enters the labor force. Individuals of generation $t$ live for two periods.

4.1. Production

To produce a good, each worker supplies inelastically one unit of labor in each period. The aggregate supply of workers evolves over time at the endogenously determined rate of population growth. In the early Malthusian phase, the agricultural sector is the only operating since the industrial sector is not yet economically viable. As technology in the industrial sector grows over time, at some point the productivity threshold is reached, the industrial sector emerges, and both sectors operate in the economy.

4.1.1. Production in both sectors

The output produced in the agricultural sector occurs according to a constant-returns-to-scale technology. In period $t$, $Y^A_t$ is determined by land, $X_t$; labor employed in the agricultural sector, $L^A_t$; agricultural technology $A^A_t$, determined endogenously; and by an additional factor that measures initial land fertility $\Upsilon > 0$.

$$Y^A_t = (Y A^A_t X_t)^\alpha (L^A_t)^{1-\alpha} \quad \text{for} \quad 0 < \alpha < 1,$$  

(1)
\[ L_t^A = (1 - \lambda_t) L_t \] where \((1 - \lambda_t)\) is the share of workers in the agricultural sector. \((1 - \lambda_t) \in (0,1)\) but it is equal to one \((\lambda_t = 0)\) until the emergence of the industrial sector.

To allow for the characterization of the periods before and after the emergence of the industrial sector, and to avoid any nuisance with the performance and movements of workers to the industrial sector, the output of the industrial sector has two structures: before and after the emergence of industry.

In the pre-industrial era, we have a linear production function relying on technology \(A_t^I\) and on efficient labor \(H_t\), at each \(t\):

\[ Y_t^I = A_t^I H_t, \quad (2) \]

with \(H_t = \lambda_t h_t L_t\), where \(h_t\) is the human-capital level and again \(\lambda_t\) is the share of workers in the industrial sector.

After the emergence of the industrial sector, constant returns to scale are assumed in the production function. The same happens to technology gains. This implies that now both sectors are always open with \(\lambda_t\) always higher than zero (marginal productivities tend to infinity when number of workers tends to zero). The elements technology and efficient labor are maintained:

\[ Y_t^I = (A_t^I)^{1-\theta} (H_t)^{\theta} \quad \text{for} \quad 0 < \theta < 1, \quad (3) \]

Finally, the total labor force is given by the sum of workers in both sectors:

\[ L_t = L_t^A + L_t^I, \quad (4) \]

where \(L_t^I = \lambda_t L_t\) and \(L_t > 0\) in each period \(t\).

4.1.2. Factor prices, labor market and the technology threshold

The economy has two types of agents: workers and elites. Workers receive their wages according to their productivity in the sector they are working on. Elites receive rents from land, since they own their property rights. Thus, return to land is not zero. Property rights are not transmissible to other elite members or workers. They are inherited by the child of each member of the elite.

Rents are determined as the marginal gains for each unit of land held by an elite member. We define \(\bar{x}^i > 0\) as the share of land held by the elite member \(i\), and we assume that all members have the same share of land. Thus, the rent received by \(i\) is:

\[ \rho_t = \alpha(YA_t^A)^{\alpha} \left( X_t \right)^{\alpha-1} (L_t^A)^{1-\alpha}. \quad (5) \]
We are going to assume a fixed value for land $X_t = 1$. From above, rents are positively related with technology, land fertility and the number of workers allocated to the agricultural sector: $\rho_A(Y, A_t^A, L_t^A) > 0$, $\rho_Y(Y, A_t^A, L_t^A) > 0$ and $\rho_{L,A}(Y, A_t^A, L_t^A) > 0$ for any $Y, A_t^A, L_t^A > 0$.

Depending on the era, before or after the emergence of industrial sector, wages can be earned either in the agricultural sector or in the agricultural and the industrial sector. The market for labor is perfectly competitive and thus wages are given by the marginal productivity of labor in each sector. Given (1), the marginal product and hence the inverse demand of labor in the agricultural sector is:

$$w_t^A = (1 - \alpha)(YA_t^A)^{\alpha}(X_t)^{\alpha}((1 - \lambda_t)L_t)^{-\alpha},$$  \hspace{1cm} (6)

where $w_t^A$ is the wages of agriculture workers.

From (2), before the industrial sector rises, workers would earn the potential wage:

$$w_t^I = A_t^I h_t,$$  \hspace{1cm} (7)

In the second phase, marginal productivity is in turn determined using (3):

$$w_t^I = \theta(A_t^I)^{1-\theta}(H_t)^{\theta-1}h_t = \theta(A_t^I)^{1-\theta}(\lambda_tL_t)^{\theta-1}(h_t)^{\theta}.$$  \hspace{1cm} (8)

From (6) and (7), the productivity of the industrial sector is finite and initially low (if we consider initial low technology values for industrial technology), whereas productivity in the agricultural sector tends to infinity for low initial levels of employment. Thus, the agricultural sector is open in every period, and the industrial sector emerges only when its labor productivity exceeds the marginal productivity of labor in the agricultural sector, considering that the entire labor force is employed in the agricultural sector. When the emergence takes place, the new production structure is also applied in the industrial sector. Then, (6) and (8) must be equal to guarantee the perfect labor mobility assumption and hence determine the share of workers in each sector. To establish necessary conditions for the emergence of the industrial sector we set Lemma 1.

**Lemma 1:** If wages are determined by (6) and (7), there is a threshold value for industrial technology $\hat{A}_t$ from which the industrial sector is economically viable:

$$\hat{A}_t^I > \frac{(1 - \alpha)(YA_t^AX_t)^{\alpha}}{L_t^A h_t}.$$  

See proof in the appendix.
When the threshold is exceeded the industrial sector emerges, and $\lambda_t$ is no longer zero. From Lemma 1, if $A_t < \tilde{A}_t$ then the agricultural sector is the only open sector and so wages are set equal to the marginal product of the agricultural sector $w_t = w_t^A$. Otherwise, if $A_t \geq \tilde{A}_t$, by the perfect mobility of workers, marginal products equalize $w_t = w_t^A = w_t^I$ and wages are set to be equal to the marginal product of the industrial sector (8). The equilibrium share of labor between the two sectors at period $t$ is given by:

$$\lambda_t = \begin{cases} 0 & \text{if } A_t < \tilde{A}_t \\ \frac{\theta \tilde{A}_t^\frac{1}{\theta} L_t^\frac{1}{\theta}}{Y_A^\frac{1}{\theta} X_t (1-\alpha)^{\frac{1}{\theta}} + \theta \tilde{A}_t^\frac{1}{\theta} (h_{t-1})^\theta} & \text{if } A_t \geq \tilde{A}_t. \end{cases}$$

(9)

And,

$$w_t = \begin{cases} (1-\alpha) (Y A_t^A)^\alpha (X_t)^\alpha ((1-\lambda_t) L_t)^{-\alpha} & \text{if } A_t < \tilde{A}_t \\ \theta (A_t^I)^{1-\theta} (\lambda_t L_t)^{\theta-1} (h_t)^{\theta} & \text{if } A_t \geq \tilde{A}_t. \end{cases}$$

(10)

### 4.2. Workers

As for workers, they are raised by their parents in the first period of their lives (childhood) and may be educated, acquiring human capital. In the second period of their lives (adulthood), individuals supply their efficiency units of labor and allocate the resulting wage income. The preferences of members of generation $t$ (those born in $t-1$) are defined over consumption above a subsistence level $\tilde{c} > 0$ in and over the potential aggregate income of their children; i.e., the number of their children, their acquired human capital and their correspondent wages (observed in $t+1$). They are represented by the utility function:

$$u_t = c_t^\gamma (h_{t+1} n_t)^{1-\gamma} \text{ for } 0 < \gamma < 1,$$

(11)

where $c_t$ is consumption, $h_{t+1}$ is the human-capital level of each child and $n_t$ is the number of children of members of generation $t$. Following Galor and Weil (2000), the individuals function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that ensure that, for sufficiently high income, there exists an interior solution for the utility maximization problem. For a sufficiently low level of income the subsistence consumption constraint is binding. Let $z_t = w_t h_t$ (where for $e_t = 0, h_{t+1} = 1$) be the level of potential income per worker, which is divided between expenditure on

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Note that for the easy tractability of the equilibrium we assume that $\theta = 1 - \alpha$. 

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child-rearing (quantity as well as quality) and consumption. We will define \( \bar{z} \) as the level of potential income below which subsistence consumption is binding.

Let \( \tau^e > 0 \) be the time endowment cost faced by a member of generation \( t \) for raising a child, regardless of quality, and let \( g(\tau^e, T_t) > 0 \) be the time endowment cost necessary for each unit of education per child. The function \( g(\cdot) \) depends positively on \( \tau^e > 0 \), a fixed cost of educating children, and negatively on \( T_t \geq 0 \). It depicts the intervention of elites on the process of education. \( T_t \) is the amount of taxes (i.e., resources) raised by elites among themselves to reduce the cost of education to incentive parents (workers) to educate their children. \( T_t = f(t_t, b_t) \), which relies positively on both variables, tax rate \( (t_t) \) and elites’ bequest \((b_t)\) — see section 4.3 below.

Regarding human capital in the second period of life, it is determined by the units of education received during childhood. The human capital is an increasing and concave function of education. The more education the higher the human capital but the gains associated to each additional unit have diminishing returns\(^4\).

\[
h_{t+1} = h(e_{t+1}),
\]

where \( h(0) = 1, \lim_{e \to \infty} h'(e_{t+1}) = 0, \lim_{e \to 0} h'(e_{t+1}) = \chi < \infty \). In the absence of education, individuals possess basic skills — one efficiency unit of human capital.

We can now sketch the budget constraint faced by parents in the second period:

\[
c_t + w_t h_t n_t (\tau^r + g(\tau^e, T_t)e_{t+1}) \leq w_t h_t,
\]

4.2.1. Optimization

The members of generation \( t \) maximize the utility subject to the budget constraint. They choose the number of children and the level of education of each child and their own consumption. Substituting (13) into (11), the optimization problem for a member of generation \( t \) reduces to:

\[
(n_t, e_{t+1}) = \arg\max \{w_t h_t (1 - n_t (\tau^r + g(\tau^e, T_t)e_{t+1}))\}^{\gamma} \{(h_{t+1}n_t)\}^{1-\gamma},
\]

subject to

\[
w_t h_t (1 - n_t (\tau^r + g(\tau^e, T_t)e_{t+1})) \geq \bar{c} \]

\[
n_t, e_{t+1} \geq 0
\]

It follows from the optimization process:

\(^4\) We follow, for instance, Galor et al. (2009) and Galor and Moav (2006), although there are other interesting approaches, such as linking human capital to the growth rate of technology or linking it to teacher’s wages although this out of the scope of the paper.
For a binding consumption constraint \( z_t < \bar{z} \), the optimal number of children for a member of generation \( t \) is an increasing function of individual \( t \)'s income. This mimics one of the fundamental features of the Malthusian era. The individual consumes the subsistence level \( \bar{c} \), and uses the rest of the time endowment for child-rearing. The higher the wage he earns, the lower the time he allocates to labor so that the time spent rearing his children increases.

Independently of the division between time devoted to consumption and child rearing, the units of education for each child only depend on the relative weight of raising costs and educating costs. While the raising costs are constant, the educating costs depend on the willingness of elites to devote resources to foster education. The higher the resources devoted to education by elites the higher the units of education given to children. Using (14) and (15), the optimization with respect to \( e_{t+1} \) shows this as the implicit function \( E(.) \) only depends on \( e_{t+1} \) and \( T_t \):

\[
E(e_{t+1}, T_t) = h'_{t+1}(\tau^r + g(\tau^e, T_t)e_{t+1}) - h_{t+1}g(\tau^e, T_t),
\]

where \( E'_e(e_{t+1}, T_t) < 0 \). \( E'_r(e_{t+1}, T_t) = g'(\tau^e, T_t)[h'_{t+1}e_{t+1} - h_{t+1}] > 0 \) for a specific set of equations. To guarantee that for a positive level of \( T_t \) the chosen level of education is higher than zero, it is assumed that:

\[
E(0,0) = h'_{t+1}(0)\tau^r - h_{t+1}(0)g(\tau^e) = 0,
\]

**Lemma 2:** If (A 1) is satisfied then, for the specific set of equations referred above, the level of education of generation \( t \) is a non decreasing function of \( T_t \).

\[
e_{t+1} \begin{cases} = 0 & \text{if } T_t \leq 0 \\ > 0 & \text{if } T_t > 0 \end{cases} \quad \text{and} \quad e'_{t+1}(T_t) > 0 \quad \text{for } T_t > 0
\]

See proof in the appendix.

From the above information and (15) we can draw some conclusions on the behavior of education and the number of offspring.

**Proposition 1:** From Lemma 2, (15), (16) and (A 1):

(A) The number of offspring and level of education are affected by the level of \( T_t \).

An increase of \( T_t \) results in a decline in the number of offspring and in an increase in their level of education: \( \frac{\partial n_t}{\partial T_t} < 0 \) and \( \frac{\partial e_{t+1}}{\partial T_t} > 0 \).
(B) The number of offspring is affected by changes in the potential income of parents if the subsistence consumption constraint is binding, while the level of education is not affected. Otherwise, none of the two variables are affected:

\[
\begin{align*}
\frac{\partial n_t}{\partial z_t} &> 0 \quad \text{and} \quad \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if} \quad z_t < \bar{z} \\
\frac{\partial n_t}{\partial z_t} &\leq \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if} \quad z_t \geq \bar{z}
\end{align*}
\]

4.3. Elites

As already stated, elites have one child each. There are no decisions on the quantity or quality of children by elites. In the first period (childhood) of their lives they are raised by their parents. They receive their bequest and decide how to spend it. Namely, they can use part of their bequest to support education or use it to consume and leave a bequest in adulthood to their children. Hence, in the second period of their lives (adulthood), elites divide the value of rent from land and the bequest left from period one into consumption and the bequest to their children.

The preferences of members of generation \( t \) (those born in period \( t - 1 \)) are defined over consumption as well as over the bequest left to their children. They are represented by the utility function:

\[
u_t = c_{t+1}^\mu (b_{t+1})^{1-\mu} \quad \text{for} \quad 0 < \mu < 1,
\]

where \( c_{t+1} \) is consumption in period two, \( b_{t+1} \) is the bequest for the child. The elites function is again strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions. For the sake of simplicity we will assume that \( \mu = \gamma \).

The available income to use as consumption and bequest is defined by the period two rent and the remains of the bequest after taking the amount to support education. This amount \( (T_t) \) is taken in the first period, when the bequest is received. Bequest keeps the same value through periods – interest rate equals zero. \( T_t = f(t_t, b_t) \) where \( f_t(t_t, b_t) > 0 \) and \( f_b(t_t, b_t) > 0 \). We will assume \( f_{tt}(t_t, b_t) < 0 \) and \( f_{bb}(t_t, b_t) < 0 \) since a concave reaction of the amount spent in decreasing the cost of education appears to be reasonable. A boundary to the gains of more spending seems to be plausible to avoid infinite gains. The rent depends on the amount of land each elite member has. \( \bar{x}^i \) establishes how land is divided among elites; the higher the value the less dispersed is land.
We can now sketch the budget constraint faced by elites in the first period:
\[ c_{t+1} + b_{t+1} \leq \bar{x}^i \rho_{t+1} + (1 - t_t)b_t, \]  
(18)

**4.3.1. Optimization**

Members of generation \( t \) maximize the utility subject to the budget constraint. They choose the tax level, the consumption in next period and the next period bequest for their children. Substituting (18)(13) into (17), the optimization problem for a member of generation \( t \) reduces to:

\[
(t_t, b_{t+1}) = \underset{\text{argmax}}{\{x^i \rho_{t+1} + (1 - t_t)b_t - b_{t+1}\}} \{h_{t+1}n_t\}^{1-\gamma},
\]  
(19)

subject to

\[
b_{t+1} \geq 0
\]

and \( t_t \in [0,1] \)

It follows from the optimization process:

\[
b_{t+1} = (1 - \gamma)(\bar{x}^i \rho_{t+1} + (1 - t_t)b_t),
\]  
(20)

Elites spend \((1 - \gamma)\) of their income in giving a bequest to their children.

Using (5) and (19), the optimization with respect to \( t_t \) shows this as an implicit function \( G(\cdot) \) which depends on \( t_t, Y, A_t^A, A_t^I \) and \( L_t \):

\[
G(A_t^A, Y, A_t^I, L_t, t_t) = \bar{x}^i \frac{d\rho_{t+1}}{dt_t} - b_t = 0,
\]  
(21)

The implicit function has two different characterizations depending on if we are before or after the structural break (see below). There are different effects that influence the decision of elites and are reflected in \( G(\cdot) \). The primary effect is the “bequest effect”. The higher the bequest, the higher is the amount transferred to support education. This effect is always negative. The “rent effect” is the other effect that can be divided between two main effects: “technology effect" and “workers effect". The technology effect is positive since a higher amount spent in supporting education increases the externality of the industrial technology on the agricultural technology. The sign of the worker effect is ambiguous. With more education: agriculture share can increase due to the externalities that increase productivity in this sector, while the industrial sector can benefit from more human capital that also increases marginal productivities in this sector. Thus, depending on the strength of forces, either the share of workers in industry or in agriculture may increase. Now, since elites’ decision relies on an implicit function
and \( t_e \in [0,1] \), we can draw some conclusions on the decision, depending on whether \( G(\cdot) \leq 0 \).

**Lemma 3:** The decision to set taxes higher than zero depends on the value of the implicit function. Since \( G(\cdot) \) can take different values in \([0,1]\), then:

If for the entire interval \([0,1]\):

\[
\begin{align*}
G(\cdot) < 0, & \quad \Rightarrow \quad t_e = 0 \\
G(\cdot) > 0, & \quad \Rightarrow \quad t_e = 1
\end{align*}
\]

If in the interval \([0,1]\):

\[
G(\cdot) = 0, \quad \Rightarrow \quad t_e \in [0,1]
\]

See proof in the appendix.

Thus, when \( G(\cdot) \) does not equalize to zero then either an increase of \( t_e \) increases the utility at a maximum of \( t_e = 1 \) or it decreases the utility so that the best choice is \( t_e = 0 \). Otherwise, \( t_e \in [0,1] \), so that the best choice is an interior solution.

From (5), (6), (7) and (21) we can determine the implicit function before the structural break:

\[
G(\cdot) = \frac{\alpha x^1(L_t) \xi(A_t^b)^\delta}{(1 + e_{t+1})^{-\frac{\beta(1-a)}{a}}} \left[ \frac{(1 - a)^{\frac{1}{2}}}{((1 + h_t L_t^2)^{\xi(A_t^b)} A_t^1)^{\frac{1}{2}}} \right]^{1-a} \frac{de_{t+1}}{dt} \left( (A_t^b)^b - \frac{\beta(1-a)(1 + e_{t+1}(A_t^b)^b)}{\alpha (1 + e_{t+1}^b)} \right) - b_t, \tag{22}
\]

where only \( e_{t+1} \) depends on \( t_e \). Before the break, the decision of elites on education relies on the technology effect, which is higher than the workers effect and the bequest effect.

**Lemma 4:** Before the structural break, for \( G(\cdot) \geq 0 \) the condition \( (A_t^b)^b - \frac{\beta(1-a)(1 + e_{t+1}(A_t^b)^b)}{\alpha (1 + e_{t+1}^b)} \) must be positive.

Proof: follows directly from (22).

After the structural break, when industrial revolution takes off, the decision rule differs due to the new industrial structure. The decision continues to contemplate the same three effects and one more. In contrast with the previous decision rule, the “population effect” now does not vanish, and thus the rent effect has now three effects within it. The population effect relates to education since the more education the less time endowment workers have to raise children so that the quantity–quality trade-off is instrumental. The implicit function after the structural break is derived from (5), (6), (8) and (21) – see appendix for the full equation.

---

5 We show by simulation that \( G(\cdot) \) is a decreasing function with respect to \( t_e \).
Lemma 5: After the structural break, for $G(.) \geq 0$ it must be true that:

$$
1 - (1 - \alpha) \frac{(L_t)^{\delta} (A_t^A)^{\delta} X_{t+1} (1-\alpha)^{\frac{3}{2}} g(1 + h_t t^2)^{\delta} A_t^A (1+t+e_t+1)^{\beta}(\beta(1+t+e_t+1)^{\beta})}{(L_t)^{\delta} (A_t^A)^{\delta} X_{t+1} (1-\alpha)^{\frac{3}{2}} g(1 + h_t t^2)^{\delta} A_t^A (1+t+e_t+1)^{\beta}(\beta(1+t+e_t+1)^{\beta})} \text{ is positive.}
$$

Proof: follows directly from implicit function condition shown in the appendix.

Regarding the decision rules, elites define when they support and allow education on the economy. Decisions depend on the macroeconomic environment and main variables, such as land fertility and agricultural technology. These implications are analyzed in section 4.2.

4.4. Dynamical Paths

The economy is governed by three main macroeconomic variables: agricultural productivity $A_t^A$, industrial productivity $A_t^I$ and the path of working population $L_t$.

4.4.1. Population Dynamics

From (15), the size of the labor force in period $t + 1$, $L_{t+1}$, is determined by:

$$
L_{t+1} = L_t n_t = \begin{cases} 
1 - \frac{\tilde{c}}{\tilde{W}_t} \frac{L_t}{(r^r + g(t^e, t^e) e_{t+1}) L_t} & \text{if } z_t < \bar{z}, \\
1 - \frac{\gamma}{\tilde{W}_t} \frac{L_t}{(r^r + g(t^e, t^e) e_{t+1}) L_t} & \text{if } z_t \geq \bar{z}
\end{cases}
$$

(23)

where the initial historical size of the adult population, $L_0 > 0$, is given.

4.4.2. Technology Dynamics

The level of each technology is affected by its level in the previous period. Agricultural technology at time $t + 1$ is affected by two elements: the externality of the “learning by doing effect” and general knowledge effect of population in technology; and the external effect from the gains of educating the youngsters in the period jointly with the existent level of industrial technology. This latter effect allows for interconnections between technology and education and existent working population and level of agricultural technology. The law of motion of agricultural technology is such that:

$$
A_{t+1}^A = (1 + e_{t+1} (A_t^I)^b) (L_t)^c (A_t^A)^\delta,
$$

(24)
where \((L_t)^\varepsilon(A_t^\delta)^\delta\) captures the “learning by doing effect” and general externalities of growing population in agricultural technology. The factor \(e_{t+1}(A_t^b)(L_t)^\varepsilon(A_t^\delta)^\delta\) is the external effect of industrial technology and education.

We assume that \(\varepsilon > 0\) and \(\delta > 0\) and \(\varepsilon + \delta < 1\), which implies that population has decreasing effect in knowledge creation, and it also implies a "fishing out" effect, namely the negative effect of past discoveries on making discoveries today. In addition, \(b > 0\) so that when people are educated, externalities of industrial technologies are spilled to technology in agriculture.

Evolution in industrial technology is given by the past period level of technology and the improvement of knowledge driven by working population size measured by its human capital. The more human capital and the more the number of workers in the economy, the more the gains to industrial technology driven by learning by doing and externalities associated with human capital.

\[ A_{t+1}^I = (1 + h_t L_t^\zeta) A_t^I, \]  

Equation (25) shows that industrial technology advances according to the expansion of the existent level of technology due to increasing population and human capital, but in a diminishing returns fashion.

The initial historical levels of both technologies, \(A_0^I, A_0^I > 0\), are given.

5. **Dynamics of the Development Process**

In this section, it is examined how the structure of the economy and agents’ decisions shape the path of the development process. It is shown how the economy can evolve from a pre-industrial equilibrium to a state of sustained economic growth and how land fertility, agricultural technology and elites’ decisions affect the economic equilibrium during the different states. It is observed that countries with higher land fertility have elites more open to support education. Moreover, more positive shocks in agriculture technology due to a presumable Agricultural Revolution also cause the early emergence of education. The timing of the demographic transition is then a consequence of elites’ decisions on education.

5.1. **Before the Industrial Revolution**

This section studies the transition from an agricultural to an industrial economy, i.e., it is described the evolution of the economic system within the
Malthusian era and the endogenous transition to industrialization. It is shown that the transitional process is straightforward but will depend on the initial level of land fertility, agricultural technology and on the elites’ decision on supporting education. Hence, these causes also affect the rise of education in the economy delaying the process of deep industrialization.

During the Malthusian era, the economy is governed by the dynamical system given by equations (23), (24) and (25) which yield the sequence of state variables \( \{A_t^A, A_t^I, L_t\}_{t=0}^\infty \) given the initial values \( (A_0^A, A_0^I, L_0) \).

Following Ashraf and Galor (2011), the pre-industrial equilibrium can be analyzed by the behavior of the two variables \( A_t^I, L_t \) and the distance to the Industrialization frontier. The industrial technology variable does not affect the pre-industrial equilibrium since until the emergence of the industrial sector it is just a latent variable. It may only interfere on the elites’ decisions but, as shown in the simulations below, it does not affect them before the take-off of the industrial sector. It must also be stressed that in the pre-industrial era the economy is under the Malthusian regime, i.e., the economy evolves under the assumption that the subsistence consumption constraint is binding and so fertility depends on income of workers. Thus, as it is shown, under a steady state equilibrium the economy is trapped on the Malthusian regime and on a binding consumption constraint.

5.1.1. The Industrialization Frontier

The Conditional Industrialization Frontier (CIF) gives the frontier between the agricultural economy and the industrial economy. It is a geometric locus, in \( \{A_t^A, L_t\} \) space, for a given \( A_t^I \) where workers are indifferent between working or not in the industrial sector. Once the economy’s trajectory crosses the frontier, the industrial sector becomes operative. The CIF is then given by:

\[
CIF|A^I_t \equiv \{(A^A_t, L_t): L_t = \hat{L}(A^A_t, A^I_t)\},
\]

and we can establish the following lemma:

**Lemma 6:** If \( \{A^A_t, L_t\} \) belongs to the CIF then, for a given \( A^I_t \),

\[
L_t = \frac{(1 - \alpha)\bar{Y}X_t}{(h_tA_t^I)^\frac{1}{\alpha}}
\]

where \( \frac{\partial L(A^A_t, A^I_t)}{\partial A_t^A} > 0 \) and \( \frac{\partial L(A^A_t, A^I_t)}{\partial A_t^I} < 0.\)
Proof: Follows directly from Lemma 1, (6), (7) and (26).

The CIF is upward sloping. In the region strictly below the frontier, agriculture is the only open sector, whereas in the region above both sectors are open. The higher $A^t$, the closer we are from the trigger and from surpassing the CIF.

For the case of the agricultural technology locus, we set it for all the pairs $\{A^t, L_t\}$ such that $A^t$ it is in steady state.

$$AA \equiv \{(A^t, L_t): A^t_{t+1} - A^t_t = 0\}, \quad (27)$$

**Lemma 7:** If $\{A^t, L_t\}$ belongs to $AA$ then,

$$L_t = (A^t)^{1-\delta} \equiv L^{AA}(A^t)$$

where $\frac{\partial L^{AA}(A^t)}{\partial A^t} > 0$ and $\frac{\partial^2 L^{AA}(A^t)}{\partial(A^t)^2} > 0$.

**Proof:** Follows from (24) and (6) using the steady state condition and (27).

The $AA$ locus is a convex, upward sloping curve. Above $L^{AA}$ the number of workers is large enough to ensure the expansion of the technology frontier, overcoming the erosion effects of imperfect intergenerational transmission of knowledge. Below the $L^{AA}$ workers are too few to overcome the latter effect, shrinking the technology level.

The population locus ($LL$) is the set of all pairs $\{A^t, L_t\}$ such that $L_t$ is in steady state$^6$, regarding that the CIF is not surpassed and considering the subsistence consumption constraint. Since the Malthusian era is characterized by a direct interconnection between fertility and income, the population locus should then be considered when the subsistence consumption constraint is binding.

$$LL \equiv \{(A^t, L_t): L_{t+1} - L_t = 0\mid L_t < \hat{L}; z_t < \bar{z}\}, \quad (28)$$

**Lemma 8:** If $\{A^t, L_t\}$ belongs to $LL$ then,

$$L_t = \left[\frac{(1 - \tau^*(1 - \alpha)}{\hat{c}}\right]^{\frac{1}{\beta}} Y A_t X_t \equiv L^{LL}(A^t)$$

where $\frac{\partial L^{LL}(A^t)}{\partial A^t} > 0$ and $\frac{\partial^2 L^{LL}(A^t)}{\partial(A^t)^2} = 0$.

**Proof:** Follows from (23) using the steady state equilibrium condition and (28).

---

$^6$ Although population has increased in during some periods of the Malthusian era (Maddison, 2003), the model regards this period in a steady state level.
Hence, the $LL$ locus is an upward sloping linear function. $L_t$ grows over time below the $LL$ locus ($L_{t+1} > L_t$) when for a lower population size wages increase and, hence, allows for fertility above replacement. Otherwise, wages are lower and, due to the consumption constraint, resources available for fertility are reduced ($L_{t+1} < L_t$). The relationship between the $LL$ locus in Lemma 8 and the $CIF$ in Lemma 6 is:

**Lemma 9:** For $A_t^i > 0$ and for all $A_t^A$ such that $(A_t^A, \tilde{L}(A_t^A, A_t^i)) \in CIF|A_t^i$ and $(A_t^A, L^{LL}(A_t^A)) \in LL$

$$\tilde{L}(A_t^A, A_t^i) \gtrless L^{LL}(A_t^A) \quad \text{if and only if} \quad A_t^i \gtrless \frac{\bar{e}}{1-(1-\tau^r)h_t}$$

Proof: Follows from comparing $\tilde{L}(A_t^A, A_t^i)$ and $L^{LL}(A_t^A)$ in Lemma 6 and Lemma 8, respectively.

Hence, for $A_t^i < \frac{\bar{e}}{1-(1-\tau^r)h_t}$ the $CIF$ is above the $LL$ locus. Nevertheless, the more $A_t^i$ increases the closer is the trigger of the industrial sector and when $A_t^i = \frac{\bar{e}}{1-(1-\tau^r)h_t}$ the $CIF$ equalizes the $LL$ locus. After this point, $A_t^i > \frac{\bar{e}}{1-(1-\tau^r)h_t}$ the $CIF$ is below the $LL$ locus – the industrial sector emerges.

### 5.1.2. Equilibrium and Global Dynamics

If we consider the pre-industrial Malthusian equilibrium, we have to assure that the condition $A_t^i < \frac{\bar{e}}{1-(1-\tau^r)h_t}$ in Lemma 9 verifies and the subsistence consumption constraint is binding $z_t < \tilde{z}$. Following these conditions, the Malthusian steady state is characterized by a globally stable steady state equilibrium $\{A_{ss}^A, L_{ss}\}$. Using Lemma 7 and Lemma 8 the pre-industrial steady-state values of productivity in the agricultural sector, $A_{ss}^A$, and the size of working population, $L_{ss}$, are given by:

$$A_{ss}^A = \left[\frac{(1-\tau^r)(1-\alpha)}{\bar{e}}\right]^{\frac{\bar{e}}{\alpha(1-\delta-\epsilon)}} \left[\frac{1-\delta}{\bar{e}}\right]^{\frac{\bar{e}}{(1-\delta-\epsilon)}}$$ \(\text{(29)}\)

$$L_{ss} = \left[\frac{(1-\tau^r)(1-\alpha)}{\bar{e}}\right]^{\frac{\bar{e}}{\alpha(1-\delta-\epsilon)}} \left[\frac{1-\delta}{\bar{e}}\right]^{\frac{\bar{e}}{(1-\delta-\epsilon)}}$$ \(\text{(30)}\)

By ruling out the unstable equilibrium at the origin ($L_0 > 0$ and $A_0^A > 0$), the globally stable equilibrium $\{A_{ss}^A, L_{ss}\}$ is maintained. At initial stages of development, agriculture is the pervasive sector since the latent industrial sector
has a very low level of productivity and so it is not sufficiently attractive. The economy operates exclusively in the agricultural sector. So, the CIF locus is located above the LL locus and the above mentioned dynamics of $L_t$ and $A_t^A$ are valid.

To guarantee that the discrete dynamical system is globally stable and that the convergence to the steady state takes place monotonically over time, the following lemma is considered:

**Lemma 10:** If $A_t^I < \frac{\bar{c}}{(1-\bar{r})h_t}$ then the equilibrium in the dynamical system:

1. is globally stable if the Jacobian matrix $J(A_{ss}, L_{ss})$ has real eigenvalues with modulus less than 1;
2. and the convergence to the steady state is monotonically stable.

See proof in the appendix.

The economy is initially in an early stage of development meaning that the economy evolves in the pre-industrial regime and both variables $(A_t^A, L_t)$ gravitate at the steady state values. To guarantee that the pre-industrial equilibrium remains until the emergence of the latent industrial sector, we must assure that the subsistence consumption constraint remains binding during this regime so that:

$$z_t \mid A_{ss}^A, L_{ss} = w_{ss} h_t < \bar{z}, \quad (A\,2)$$

For an initial $h_t = 1$. With only the agricultural sector operative, all workers are employed in this sector, thus, it follows from (1) that the steady-state level of income per worker is given by:

$$y_{ss} = (YA_{ss}^A X)^{\alpha}(L_{ss})^{-\alpha}, \quad (31)$$

Using (29) and (30), the steady-state level of income per worker is in line with the dynamics under the Malthusian era - in the long-run, the level of income is independent of the level of technology and it is constant, $\tilde{y}_{ss} = 0$. Note that income per capita is also affected by the level of natural land endowment of countries, thereby implying different long-run levels of income per capital across countries.

5.2. **The Industrial Revolution**

As the economy evolves under the Malthusian era within the pre-industrial steady state, it operates exclusively in the agricultural sector. The latent and endogenous process of industrialization implies the take-off to a state of sustained economic growth in a near future. This section examines the transition from the
Malthusian regime, through the Post-Malthusian Regime, to the demographic transition and Modern Growth era.

5.2.1. Dynamics

This section describes the two different potential regimes after the emergence of the industrial sector. It is assumed that under the pre-industrial period the subsistence consumption constraint is binding. Thus, the economy is still under the Malthusian regime and it is trapped since there is a stable steady-state equilibrium that dismisses any chance of moving out of that trap - see (A 2). However, after the emergence of the industrial sector, in the first regime the subsistence consumption constraint is still binding. Despite of this, since we are no longer under the steady state equilibrium of the previous regime, wages increase and so the subsistence constraint will vanish in time. The Post-Malthusian era starts and population grows faster although income still has an effect on fertility. As for the second regime, the subsistence constraint is no longer binding. This means that there will be no direct effect of income on fertility. Population grows at a constant level that will only be affected by choices of workers on education due to elites’ decision of supporting education – see (23).

In the first regime the economy will be governed by a four-dimensional non-linear first-order autonomous system:

\[
\begin{align*}
A_{t+1}^A &= (1 + e_{t+1} (A_t^I)^b) (L_t) e^e (A_t^A)^{\delta} \\
A_{t+1}^I &= (1 + h_t L_t^{\delta})^{\gamma} A_t^I \\
e_{t+1} &= e(T_r (A_t^A, A_t^I, L_t)) \\
L_{t+1} &= \frac{1 - \tilde{c}}{\tilde{w}_t} \left( \tau^r + g(\tau^e, \tau^r) e_{t+1} \right) L_t
\end{align*}
\]

for \( z_t < \tilde{z} \) (32)

In the second regime, since the subsistence consumption constraint is no longer binding, the regime is governed by the same four-dimensional system although now population growth does not depend on income of workers:

\[
\begin{align*}
A_{t+1}^A &= (1 + e_{t+1} (A_t^I)^b) (L_t) e^e (A_t^A)^{\delta} \\
A_{t+1}^I &= (1 + h_t L_t^{\delta})^{\gamma} A_t^I \\
e_{t+1} &= e(T_r (A_t^A, A_t^I, L_t)) \\
L_{t+1} &= \frac{1 - \gamma}{\tau^r + g(\tau^e, \tau^r) e_{t+1}} L_t
\end{align*}
\]

for \( z_t \geq \tilde{z} \) (33)
In these regimes an analytical analysis is not straightforward since there are four interconnected differential equations. Nevertheless, some inferences can be drawn on the passage from one to another regime. Namely, the transition between the two regimes is given by the distance to the Malthusian Frontier \(MF\). As explained previously in (32) and (33), the economy departs from the first regime when potential income \(z_t\) exceeds that level. So, \(\{A_t^A, A_t^I, L_t, e_{t+1}\}\) belongs to \(MF\) if

\[
MF \equiv \left\{(A_t^A A_t^I, L_t, e_{t+1}) : \theta (A_t^I)^{1-\theta} (\lambda_t (A_t^A, A_t^I, L_t)L_t)^{\theta-1} (h_t)^{\theta} = \frac{\zeta}{1-\gamma} \right\}, \tag{34}
\]

**Lemma 11:** The economy surpasses the Malthusian regimes if:

\[
w_t' = \theta (A_t^I)^{1-\theta} (\lambda_t (A_t^A, A_t^I, L_t)L_t)^{\theta-1} (h_t)^{\theta} \geq \frac{\zeta}{1-\gamma}
\]

Proof: Follows from (8), definition of \(z_t\) and \(\bar{z}\), and (34).

6. **From Malthusian era to Modern Growth era**

The economy evolves from the Malthusian era to Modern Economic Growth era passing through the Post Malthusian era and the demographic transition. This path derives from section 4.1 and the two regimes explained above.

Consider an economy trapped in the pre-industrial equilibrium. Population is quite small and agricultural technology is stagnant. Concerning the macroeconomic variables, there is a globally stable steady state equilibrium on agricultural technology and population (Lemma 10). Initially education is not supported by elites and thus workers do not provide it. There is only the latent industrial sector whose productivity is growing slowly over time. Income per capita is constant as well. This is the typical Malthusian stagnation regime. Under Lemma 9 and Lemma 10 we can characterize this globally stable conditional steady state equilibrium and characterize the moment when the take off takes place. As for education, from Lemma 4 and equation (22) we know that only if these conditions are satisfied elites have incentives to support and allow education. They are only verified if \(G(\cdot)|A_{ss}, L_{ss}, A_t^I \geq 0\)

As productivity grows in the latent sector and, later on, the emergence of the industrial sector occurs, the economy changes to the Post-Malthusian regime. Now workers split between the two sectors, and with the structural break there are always workers in both sectors. As the equilibrium conditions change, population,
as well as agricultural technology, starts to grow over time. From (24) and (25), growing population has a scale effect on both technologies and there is an interconnection between variables since, now, more population and technology lead to higher wages. As income increases, and the economy still is in a Post-Malthusian regime, it affects positively fertility. More income means higher fertility and there is a boost in population. The three state macroeconomic variables grow over time. Therefore, the more income available the less restrictive is the budget constraint so that consumption increases over time. In reaction to increasing disposable income the subsistence consumption constraint vanishes. As this occurs the economy moves to the second regime. Here, population grows at a steady rate (given in (33)) and income does not affect fertility. Fertility is, then, only dependent on the quality-quantity trade-off. Elites decide again if there is education or not (see Lemma 2 and Proposition 1). From section 3.3 we know that only if $G(.) \geq 0$ and, hence, if Lemma 5 applies, elites support education. Since industrial technology is growing and the share of workers is mostly in the industrial sector, the marginal gains from the technology effect exceed at some point the workers effect and the population effect (if negative). As this condition applies, the overcoming of the bequest effect, which is negatively affected by growing industrial technology, soon follows. At this point elites have an incentive to promote education since they gain more from technology improvements than they lose from transfers of a share of their bequest. Besides the gains on technology from education, which enhance industrial and agricultural technology, the outcome of this decision is a demographic transition. As it was explained above fertility, now, only depends on the quality-quantity trade-off derived from workers’ decisions. Therefore, the more educated children is the lower is the number of offspring, causing the decrease of population growth rates.

The rise of the industrial sector and the posterior rise of education have a virtuous effect. The industrial sector allows workers to earn more and to have more resources available to allocate to children quantity and quality. When education is allowed by elites the amount earlier allocated just to quantity now splits, also being allocated to quality decreasing population growth rate but increasing technology and, hence, increasing productivity levels of workers in both
sectors, from (24) and (25), which increases earnings, implying more income available for consumption.

From simulations of the model, to be presented in the next subsection, as the economy evolves, the main macroeconomic variables take a constant behavior: population continues to grow at a small rate, productivity in both sectors increase over time with industrial productivity growing more than agricultural productivity. As for the share of workers, a shift of most of population to the industrial sector is observed. As for education, it increases over time but it remains almost stable after the initial boost.

Now it must be understood the interaction between the features referred at the introduction. Firstly, it is showed how the model behaves by itself and how elites behave in their willingness to allow the provision of education to children and consequently cause the demographic transition. Then, the relationship between land endowments and the onset of industry and education is examined to verify how they affect the industrial take off and the elites’ decisions on education. Finally, the possible role of Agricultural Revolution is discussed, namely how it can account for the onset and continuous process of industrialization and education.

From here we can draw the main hypotheses advanced in this paper:

H1 - The emergence of education and, hence, the demographic transition depends on the decision of elites: elites delay the emergence of education even after the onset of the Industrial Revolution;

H2 – Agricultural Revolution has a positive effect on the emergence of education;

H3 – Land fertility affects positively the emergence of education.

6.1. Model Calibration: Education and Demographic Transition

This section begins with the simulation of the model and its properties. Galor (2011), Lagerlöf (2006) and Voigtländer and Voth (2006) provide quantitative analyses of Unified Growth models which have similarities with this one, so their calibration of parameters will be followed closely when possible.

Firstly, some specific functional forms for human capital and the cost of education function are specified for the calibration in order to conform to Lemma 2. From (12):
\[ h_{t+1} = (1 + e_{t+1})^\beta, \]

with \( 0 < \beta < 1 \). It is an increasing, strictly concave function, of the investment on education, \( e_{t+1} \).

As for the time endowment cost necessary for each unit of education and for each child, \( g(\tau^e, T_t) \), is given by:

\[ g(\tau^e, T_t) = (\tau^e + T(t_t, b_t))^\phi, \]

where \(-1 < \phi < 0\) and \( T(t_t, b_t) = \frac{t_tc_t}{1+t_tc_t} \) is an increasing, concave function, in \( t_t \) and \( b_t \). And so, \( g(.) \) is decreasing in \( t_t \) and \( b_t \).

The baseline parameters are depicted in Table 1:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land share ((\alpha)) /Labor share ((1 - \alpha))</td>
<td>(0.4 / 0.6)</td>
</tr>
<tr>
<td>Human Capital share ((\theta))</td>
<td>0.6</td>
</tr>
<tr>
<td>Land Fertility ((\Upsilon))</td>
<td>1</td>
</tr>
<tr>
<td>Land ((X))</td>
<td>1</td>
</tr>
<tr>
<td>Division of land among elites</td>
<td>1</td>
</tr>
<tr>
<td>Weight on children in utility function ((\gamma = \mu))</td>
<td>0.645</td>
</tr>
<tr>
<td>Fixed time cost of raising children ((\tau^R))</td>
<td>0.34</td>
</tr>
<tr>
<td>Time cost of educating children ((g(\tau^e)))</td>
<td>0.119</td>
</tr>
<tr>
<td>Subsistence consumption ((\tilde{c}))</td>
<td>1</td>
</tr>
<tr>
<td>Human capital ((\beta))</td>
<td>0.35</td>
</tr>
<tr>
<td>Time endowment cost concavity ((\phi))</td>
<td>-0.9</td>
</tr>
<tr>
<td>Weight of population on agricultural “learning by doing effect” ((\varepsilon))</td>
<td>0.05</td>
</tr>
<tr>
<td>Weight of agricultural technology on agricultural “learning by doing effect” ((\delta))</td>
<td>0.07</td>
</tr>
<tr>
<td>Externality of industrial technology ((b))</td>
<td>0.80</td>
</tr>
<tr>
<td>Weight of population on industrial “learning by doing effect” ((\Delta))</td>
<td>0.05</td>
</tr>
<tr>
<td>Diminishing returns effect on industrial dynamical path ((\zeta))</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Besides this, the initial conditions of the model are given by the equilibrium values for the pre-industrial period of \( A_0^H \) and \( L_0 \) as well as for fertility, education, industrial productivity, share of workers and bequest (Table 2).

---

7 It is considered that taxes are initially zero. Since for taxes equal to zero there is no education, from (35) and (36) and Lemma 2, then \( g(\tau^e) \) must be equal to that calibrated value.
Table 2 – Initial Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population ($L_0$)</td>
<td>0.0861</td>
</tr>
<tr>
<td>Agricultural productivity ($A_0^A$)</td>
<td>0.8726</td>
</tr>
<tr>
<td>Industrial productivity ($A_0^I$)</td>
<td>0.6</td>
</tr>
<tr>
<td>Fertility ($n_0$)</td>
<td>1</td>
</tr>
<tr>
<td>Education ($e_0$)</td>
<td>0</td>
</tr>
<tr>
<td>Share of workers ($\lambda_0$)</td>
<td>0</td>
</tr>
<tr>
<td>Bequest ($b_0$)</td>
<td>1</td>
</tr>
</tbody>
</table>

Using these parameterization and initial values, the patterns of the benchmark economy resemble closely the expected patterns referred above and the patterns observed in modern history. As depicted in Figure 1 and Figure 2, initially the economy is in a pre-industrial Malthusian equilibrium with population and agricultural productivity and income per capita constant over time, while industrial productivity, keeps increasing. From both figures, it is possible to observe the take off of the industrial sector after about 40 periods.

In Figure 1, the pre-industrial Malthusian regime vanishes after the beginning of the industrial phase so that population growth rates are higher than before until the demographic transition. Consistent with the empirical evidence, this occurs after about 100 periods when education is allowed by elites and it starts increasing over time. Before, education levels were always zero. It is observed the transition from the pre industrial Malthusian regime where fertility depends on income of workers and has a positive correlation with it and where education is not provided to a Modern Growth regime where fertility no longer depends on income, education emerges after elites allow for it and the demographic transition begins. In the transition process, we have the Post Malthusian regime where fertility is higher than in the previous pre-industrial era but it still depends on income, and education is still not provided. These features are showed in Figure 1.
In Figure 2, after the pre-industrial Malthusian equilibrium, agricultural and industrial productivities increase continuously as well as population. In this process both sectors productivities have a boost due to the effect of education on them. Namely, agricultural productivity suffers a peak after education starts to be provided with high rates of growth rates in this period while industrial productivity suffers a small increase but keeps its ascending path almost constant. As time goes by, the industrial sector and income per capita continue to grow at increasing rates, although agricultural productivity grows at a slower pace than the industrial one. Figure 2 presents these features:
From the above explanations, the model and its simulations, as well as the above mentioned lemmas, it is possible to derive the following proposition.

**Proposition 2:** Elites aversion to education does not persist in time. The elites’ decision to support education by themselves occurs at some point in time. Elites may delay education but they do not prevent education to arise indefinitely.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 4, Lemma 5, (22), elites’ decision after structural break, (35) and (36).

What must be held in mind from this proposition is that, in contrast with many theoretical contributions (Mokyr, 1990; Bourguignon and Verdier, 2000; Acemoglu and Robinson, 2006; Galor et al, 2009), it is possible that elites had an incentive to allow for education. Nevertheless, and agreeing with those same theories elites also had the power to prevent education to emerge sooner due to their own choices. This means that although they decide to support education they only decide later in time which is consistent with the delayed process of education verified in history: the Industrial Revolution took place in the late 1700’s and education only spread in the middle/late 1800’s (Flora et al, 1983; Brown, 1991). This delay, although many other causes are behind them, can be traced on the power and willingness of elites to support it. If, as it is argued in the initial sections, elites were a small group of interest which had decision power on society and was the only group with enough economic resources to provide the means to educate people, they had, then, the power to inhibit or not education. Thus, they had incentives to block education right after industrial emergence, but had also incentives afterwards to support it, when it was economically beneficial to them. Complementary to the literature, it can be showed that elites are not always against education and industrial enhancement.

### 6.2. Agricultural Revolution

The effect of a possible Agricultural Revolution is now examined. The Agricultural Revolution subject is highly discussed and has not had a final outcome (Clark, 1993). The hypothesis advanced in this paper suggests that increasing technology in agriculture eases the willingness of elites to allow for education.
Higher levels of agricultural productivity mean higher initial rents available for elites during the process of industrialization. Although there is the bequest effect, which increases, now the technology effect is higher than the bequest effect sooner because gains from externalities of industry are boosted by having in the recipient technology an already higher level. In other words, the higher level of agricultural technology enhances the effect of the industrial externalities on the technology of agriculture.

From the discussion on Agricultural Revolution, there is the debate on if Agricultural Revolution really happened, and if true, when it happened. It can advanced the intuition that having shocks on agriculture productivity would cause a faster positive decision of elites on education, and a negative effect on the time of industrialization. It is possible to advance the following proposition:

**Proposition 3:** Agricultural Revolution has a positive impact on elites’ decision to educate population. The higher productivity of agriculture during the process of industrialization the more elites are prone to support education.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 1, Lemma 4, Lemma 5, (22), elites’ decision after structural break, (35) and (36).

It is shown how shocks in agriculture affect the decision of elites in supporting education as well as the small fastening on the take off of the industrial sector with higher agricultural technology shocks. It is considered a positive random shock to agriculture, using a random uniform distribution to simulate an exogenous increase in agricultural technology. It is, then, clear that higher shocks have a positive impact on the early onset of education.
Some additional insights can be added to the debate. Considering the economy is in a Malthusian equilibrium before the take-off, if the shock on agriculture productivity takes place in a moment far from the take-off, the gains vanish over time (equilibrium is globally stable) and so there is no effective impact on the outcomes in the economy. But, if there are constant shocks in the economy so that the level of population and agriculture technology increase consistently above the equilibrium levels at the time of the take off, it implies there is an effect on rents as well as on bequests and, hence, on the willingness of elites to provide education. There is a virtuous cycle in the economy that will then imply a faster economic boost due to more education and therefore higher industrial and agricultural technology growth rates. This means that countries that suffered from an Agricultural Revolution, which was mainly England and some Continental countries, but in a smaller scale, benefited from an earlier take off of education and an earlier economic boom. The other countries lagged behind which may have contributed to the divergence process in industrialization verified in this time period (Galor, 2011).

From the debate going on, it is possible to argue that there must have been agricultural technology shocks in the 1700's so that the take-off of the industrial sector and education took place earlier in countries such as England and the Netherlands. Shocks in the late 1600's or in the middle/later 1700’s may not have had the necessary impact because they have occurred too early. The argument
followed here points to the fact that a consistent level of ongoing shocks in the 1700’s period was essential to the occurrence a stronger industrial revolution and an early escape to Modern Economic Growth.

6.3. Land Fertility

The land fertility hypothesis is a byproduct of the model that runs at some stage against the existing literature (Litina, 2012). Land fertility is said to have a negative effect on the ability of countries to take off and develop. The hypothesis advanced in this paper suggests that high levels of land fertility imply a fast process of provision of education by elites. The main reason is that more land fertility decreases the risk of elites in providing education the same way as the Agricultural Revolution did - higher levels of land fertility mean higher rents available for elites during the process of industrialization. And so, the rise of education allows for a stronger externality of industrial technology on the agricultural technology. It has again to do with the balance of the bequest effect and the technology effect. The higher land fertility, the higher the rents are and the bequest, but the technology effect becomes more significant in elites’ decisions than the bequest effect in an earlier stage. As it is observed from several simulations, the onset of education is anticipated (see Figure 4):

![Figure 4: Period of emergence of education for different scenarios on land fertility](image)

From Figure 4 we can observe how indeed land fertility has a positive effect on elites’ decisions. Also there is an almost insignificant effect on the take off of the
industrial sector since with fertility of land equal to two the onset is given one period earlier while with other values the effect is null. Nevertheless, it must be stressed that this result is not a main conclusion and, from many other research studies, it is more a representation of a positive force involved in the process of growth than a definite and established fact. However, analytically and numerically one can advance the following proposition:

**Proposition 4:** Land fertility has a positive impact on elites' decision to educate population. The higher the fertility during the process of industrialization the more elites are prone to support education.

Proof: It follows from the numerical simulation of the model and can be derived from Lemma 1, Lemma 4, Lemma 5, (22), elites' decision after structural break, (35) and (36).

7. **Concluding Remarks**

The results presented in the previous sections show how interest groups can have a role on determining the pace of the economy. The relevance of these results show on one side the contribution of Unified Growth Theory to the study of the Industrial Revolution and, on the other side, the current contribution for policymakers on analyzing how developing economies face delays on their processes of development due to these political forces. Substantial research has been made on the interconnections and willingness of land elites to be interested in promoting education, mostly concluding for a negative decision (Bourguignon and Verdier, 2000; Galor et al, 2009). Nevertheless, the present paper shows that, with the right incentives, even land elites ultimately agree with the promotion of education. This result does not dismiss other theories but instead complements them. In fact, this paper aims to advance that, on the political process that opposed capitalists and landlords in the nineteenth century, due to the rising power of the former and the increasing willingness of the latter to allow education, both may have reached a confluent point where capitalists demanded education and landlords did not oppose. As for today, the lesson to take is that it important to be aware of how interest groups react and which incentives they have, in order to intervene in the best way possible and reach an agreement which is well accepted by all groups and
population in general benefit from the gains of education or any other element that may be a source of conflict between groups in the same society.

Further, the paper contributed to a deeper understanding on the role of the Agricultural Revolution and whether there was a continuous sequence of technology shocks during the eighteenth century. This is a novelty and an important contribution to the literature. The conclusion that the Agricultural Revolution may have contributed to the early onset of the Industrial Revolution and, more important, to a quicker process of education of the masses is a new highlight to the debate going on in the literature. It may then further support why England developed first than other countries from continental Europe. As for land fertility, it was possible to find this positive force underlying land endowments, which also confirms that there are positive effects deriving from land fertility rather than the commonly referred in the literature where it tends to indicate a reversal of fortunes relationship between better endowed countries and worse endowed countries (Mokyr, 1990; Litina, 2012).

Finally, the numerical simulation presents the main insights of the model and shows the main conclusions referred above. In line with Unified Growth Theory it is possible to conclude that interest groups had a role on the main events during the period of industrialization. Given their decisions, the rise of education was initially halted until it was allowed and, by consequence, the process of demographic transition occurred later in the nineteenth century. Furthermore, the event of Agricultural Revolution in the previous century positively influenced the onset of both industrialization and education.

References


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Appendix

Proof Lemma 1:

If we equalize (6) and (7), using the assumption of perfect labor mobility, we know that workers are employed also in the industrial sector if the marginal productivity in the industrial sector $A_t^I$ is equal to or exceeds the marginal productivity in the agricultural sector.

Proof Lemma 2:

Take (16), (A1) and the properties of $g(t^e, T_t)$. We know that $E(0, T_t)$ is increasing in $T_t$. Also, the $\lim_{T \to \infty} E(0, T_t)$ is higher than $E(0, 0)$ so that from (A1) it is positive. Therefore, for $T_t > 0$, and from (6), $e_{t+1} > 0$. Also, from the Implicit Function Theorem and (6), we can show that $e_{t+1}$ is a single valued function of $T_t$ and $e_{t+1} = e_{t+1}(T_t)$ so that $e'_{t+1}(T_t) = -\frac{\partial E/\partial e_x}{\partial E/\partial T} > 0$.

Proof Lemma 3:

Since we are maximizing the utility we want the values of $t_t$ that for the interval $[0,1]$ yield that maximum. It must be added also that from the numerical simulations the function $G(.)$ is always decreasing in $t_t$. So,

When $\frac{du}{dt_t} \neq 0$ for the interval of $t_t \in [0,1]$;

If $\frac{du}{dt_t} > 0 \Rightarrow G(.) = x_t \frac{d \rho_{t+1}}{dt_t} - b_t > 0 \Rightarrow t_t = 1$

If $\frac{du}{dt_t} < 0 \Rightarrow G(.) = x_t \frac{d \rho_{t+1}}{dt_t} - b_t < 0 \Rightarrow t_t = 0$

Since $\frac{du}{dt_t}$ is a decreasing function, from numerical simulations, these are the only valid cases, and $\frac{du}{dt_t} |_{t_t = 0} < 0$ and $\frac{du}{dt_t} |_{t_t = 1} > 0$ does not apply.

After-structural-break implicit function and lemma 5 explanation:
\[ G(.) = \frac{d\rho_{t+1}}{dt} = \frac{\delta^a Y(1-a) \frac{1-a}{\alpha} (L_t)^{1+a-\alpha}(A_t^b)^{\delta}}{(1 + e_{t+1}(A_t^b)) (L_t)^{\delta} (A_t^b) \delta Y X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta}} \]

\[
\begin{pmatrix}
\frac{d\varepsilon_{t+1}}{dt} n_t^{-a}(A_t^b)^{\delta} \\
1 - (1 - \alpha) \\
\end{pmatrix} = \frac{Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta}}{Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta}} \left( \frac{-1}{1 - (1 - \alpha)(1 + e_{t+1}(A_t^b)^{\delta}) n_t^{-a}(1 - \gamma)(\tau + g(.) e_{t+1})} \right) - b_t \]

Since \( b_t \) is always positive, and the derivative with respect to \( n_t \) is also always positive - these are the two latter parts of \( G(.) \) - then, if Lemma 5’s condition is negative, \( G(.) < 0 \). So, only when Lemma 5’s condition is positive will we have at some point \( G(.) \geq 0 \).

More explicitly, as \( \beta \frac{\theta}{\alpha} \) is constant, \( \frac{1}{1 + e_{t+1}(A_t^b)^{\delta}} > \frac{1}{1 + e_{t+1}(A_t^b)^{\delta}} \iff 1 > (A_t^b)^{\delta} \) which happens only when the economy is almost rural. When the economy starts to industrialize and \( (A_t^b)^{\delta} > 1 \) then, for sure we can guarantee that:

\[ 0 < \frac{Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta}}{Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta}} < 1 \]

And, hence,

\[
\begin{pmatrix}
1 - (1 - \alpha) \\
Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta} \\
Y(L_t)^{\delta} X_{t+1} (1-a) \frac{1}{\alpha} + \frac{1}{\beta \pi} (1 + h_t L_t^2) \lambda_t (1 + e_{t+1}) \beta_{\pi} (1 + e_{t+1}) (A_t^b)^{\beta} \\
\end{pmatrix} > 1 \]

So, for \( A_t^b \) sufficiently big we will have a positive condition and at some point in time \( G(.) \geq 0 \) - we will observe it in the numerical simulations.
Proof Lemma 10:

Given the Jacobian matrix:

\[
J(A^A_{ss}, L_{ss}) = \begin{bmatrix}
\frac{dA^A(A^A_{ss}, L_{ss})}{dA^A_t} & \frac{dA^A(A^A_{ss}, L_{ss})}{dL_t} \\
\frac{dL(A^A_{ss}, L_{ss})}{dA^A_t} & \frac{dL(A^A_{ss}, L_{ss})}{dL_t}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\delta & \varepsilon \left[\frac{(1 - \tau^r)(1 - \alpha)}{\tilde{c}}\right]^{\frac{1}{\alpha}} \\
\frac{\alpha \tilde{c}}{(1 - \alpha)} \left[1 - \frac{\tau^r}{\tilde{c}}\right]^{\frac{1 - \alpha}{\alpha(1 - \delta - \varepsilon)}} & 1 - (1 + \alpha)(1 - \tau^r)
\end{bmatrix}
\]

The eigenvalues are given by \(\{\lambda_1, \lambda_2\}\). We know that: \(\text{det}(A^A_{ss}, L_{ss}) = \lambda_1 \lambda_2\) and \(\text{tr}(A^A_{ss}, L_{ss}) = \lambda_1 + \lambda_2\)

\(\text{tr}(A^A_{ss}, L_{ss}) = \delta + \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} > 0\) for \((1 + \alpha)(1 - \tau^r) < 1 \iff \tau^r > \frac{\alpha}{1 + \alpha}\)

\(\text{det}(A^A_{ss}, L_{ss}) = \delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left[\frac{(1 - \tau^r)(1 - \alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1 - \delta - \varepsilon)}} + \alpha(1 - \tau^r)\)

so that the equilibrium is globally stable if: \(\lambda_1, \lambda_2 \in (-1, 1)\)

(1) To guarantee that the convergence to the steady state is monotonically stable:

i. \(\text{Det}(A^A_{ss}, L_{ss}) > 0\);

ii. and \(\text{Tr}(A^A_{ss}, L_{ss}) > 0\).

For (i):

\[
\delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left[\frac{(1 - \tau^r)(1 - \alpha)}{\tilde{c}}\right]^{\frac{\varepsilon}{\alpha(1 - \delta - \varepsilon)}} + \alpha(1 - \tau^r) > 0
\]
\[ \Leftrightarrow \delta (1 - (1 + \alpha)(1 - \tau^r)) > \varepsilon \left( \frac{(1 - \tau^r)(1 - \alpha)}{\epsilon} \right)^{\gamma(1-\delta-\varepsilon)} + \alpha (1 - \tau^r) \]

(condition 1)

**For (ii):** \(Tr(A_{ss}^A, L_{ss})\) always higher than zero from the above inequality

(2) To guarantee that the equilibrium is globally stable:

i. \(-2 < Tr(A_{ss}^A, L_{ss}) < 2;\)

ii. \(-1 < Det(A_{ss}^A, L_{ss}) < 1;\)

iii. \(Det(A_{ss}^A, L_{ss}) - Tr(A_{ss}^A, L_{ss}) \geq -1;\)

iv. and \(Det(A_{ss}^A, L_{ss}) + Tr(A_{ss}^A, L_{ss}) \geq -1.\)

**For (i):** from before we know that \(Tr(A_{ss}^A, L_{ss}) > 0 > -2\)

\[ Tr(A_{ss}^A, L_{ss}) < 2 \]

\[ \Rightarrow \delta + \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} < 2 \Leftrightarrow 1 - (1 + \alpha)(1 - \tau^r) < (2 - \delta)\tau^r \Leftrightarrow \tau^r (1 - \delta - \alpha) + 1 + \alpha > 1 \text{ P.V.} \]

\[ \Rightarrow Tr(A_{ss}^A, L_{ss}) \in (-2, 2) \]

**For (ii):**

\[ Det(A_{ss}^A, L_{ss}) > -1: \]

\[ \delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \frac{(1 - \tau^r)(1 - \alpha)}{\epsilon} \right)^{\gamma(1-\delta-\varepsilon)} + \alpha (1 - \tau^r) > -1 \text{ P.V.} \]

From condition 1 we know this inequality holds.

\[ Det(A_{ss}^A, L_{ss}) < 1: \text{ (by contradiction)} \]
\[
\delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \left(1 - \tau^r \right) (1 - \alpha) \frac{\varepsilon}{\tau} + \alpha (1 - \tau^r) \right) > 1
\]

\[\Leftrightarrow (\delta - 1)\tau^r - \alpha (1 - \tau^r) > \varepsilon \left( \left(1 - \tau^r \right) (1 - \alpha) \frac{\varepsilon}{\tau} + \alpha (1 - \tau^r) \right) \Rightarrow P.F.\]

\[\Rightarrow \text{Det}(A^A_{SS}, L_{SS}) < 1\]

**For (iii):**

\[
\delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \left(1 - \tau^r \right) (1 - \alpha) \frac{\varepsilon}{\tau} + \alpha (1 - \tau^r) \right) - \delta - \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} \geq -1
\]

\[\Leftrightarrow \alpha (1 - \delta) (1 - \tau^r) \geq \varepsilon \left( \left(1 - \tau^r \right) (1 - \alpha) \frac{\varepsilon}{\tau} + \alpha (1 - \tau^r) \right) \quad \text{(condition 2)}\]

**For (iv):**

\[
\delta \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} - \varepsilon \frac{1}{\tau^r} \left( \left(1 - \tau^r \right) (1 - \alpha) \frac{\varepsilon}{\tau} + \alpha (1 - \tau^r) \right) + \delta + \frac{1 - (1 + \alpha)(1 - \tau^r)}{\tau^r} \geq -1
\]

If condition 2 holds, then, since the trace is positive, this inequality will also hold.

**Proof proposition 3:**

To know the impact of agricultural technology on elites’ decisions we need to apply the Implicit Function Theorem on (21) so that we can derive the impact of \( A^A_t \) on \( t_e \). Computing  \( \frac{dG}{dt} \) and  \( \frac{dG}{dA^A_t} \) and since it is not possible to reach analytically a definite signal we must use a numerical simulation. What it is found is that \( \frac{dG}{dt} < 0 \) for all the period of time whereas \( \frac{dG}{dA^A_t} \) is oscillatory. Nevertheless, for the period previous to the rise of education \( \frac{dG}{dA^A_t} < 0 \). Therefore:

\[
\frac{dt_e}{dA^A_t} = -\frac{dG}{dt_e} \begin{cases} < 0 & \text{before onset of Industrial Revolution} \\ \frac{dG}{dA^A_t} > 0 & \text{after onset of Industrial Revolution} \end{cases}
\]

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So, it is inferable that the higher the value of $A_t^A$ on the onset of industrialization the more likely in that the onset of education will follow sooner. Therefore, previous improvements on agriculture (historically, during the eighteenth century) influence positively the early rise of education.